A computational platform for physically-based bank evolution and long-term meander migration

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Introduction

Since the late 1970s much research, both theoretical, experimental, and in the field, has been carried out to better understand and quantify the migration of meandering streams. This has yielded simplified analytical two-dimensional (2D) models, fully numerical 2D models, and three-dimensional (3D) models of flow, sediment transport, and channel migration. Only the analytical 2D models (such as the ones by Ikeda et al., 1981; Johannesson and Parker, 1989; and Zolezzi and Seminara, 2001) can be practically used to predict migration of an entire stream system.

In long-term river meandering models, the migration rate has been historically based on a method which relates the migration rate to the near-bank excess velocity multiplied by a dimensionless coefficient (Hasegawa, 1977; Ikeda et al., 1981)

$$M = E_0 u_b \tag{1}$$

where M is the meander migration rate, E_0 the dimensionless migration coefficient, and u_b the difference between near-bank velocity and cross-sectionally averaged velocity. E_0 is obtained by means of calibration against historic centerlines and is typically a very small number (10^{-7}-10^{-8}).

Eq. (1) is a conceptual representation of the processes responsible for bank retreat in a meander bend. Although a field study by Pizzuto and Meckelnburg (1989) lends support to Eq. (1), the erosion processes are only implicitly captured in the coefficient E_0. Its dependence on physical characteristics of the bank material and the type of bank erosion (hydraulic scour or different types of mass wasting events) is therefore unclear. Though researchers agree that E_0 is generally related to bank-soil properties and type and density of riparian vegetation, a direct relationship has not yet been established. E_0 should vary across the floodplain: however, model application typically assumes constant values.

Eq. (1) cannot adequately capture the planform evolution of natural meandering rivers, because it predicts a smooth centerline, while often planform shapes are spiky, irregular, complex (Figure 1); also, the linearity of the expression implies that the only bank retreat mechanism considered is the particle-by-particle erosion (also termed hydraulic or fluvial erosion); moreover, it does not explicitly account for local, episodic mass failure mechanisms and for an erosion threshold, and it does not consider the effect of bank geometry (since it assumes vertical sidewalls) and the impact of vertical heterogeneity of the bank materials.

The use of Eq. (1) has provided important insight into the long-term evolution of meander planform through theoretical exercises, but its use in practice has not been as successful. Most of the times, calibration of the dimensionless coefficient E_0 is challenging.

Fig. 1. Example of irregular meander planform: Mackinaw River, Illinois, USA.

With the ongoing effort in both the United States and Europe to re-naturalize highly modified streams, it cannot be expected that Eq. (1) will accurately simulate the response of meandering streams to in-stream and riparian management practices over engineering time scales, especially if there is no historical data to calibrate its coefficient. A new approach is needed that relates meander migration rates to physically-based streambank erosion and deposition rates.

Efforts are ongoing to add bank erosion physics into modeling the migration of meander bends. Kobayashi et al. (2008) and Parker et al. (2009) presented a method that relates bank retreat to the near-bank transverse sediment flux modified for slump block armoring. Few 2D numerical models have included modules for physical representation of bank erosion (Nagata et al., 2001; Duan et al., 2001; Darby et al., 2002; Rinaldi et al., 2008), usually for short reaches and simulation periods. To our knowledge, no model has tried to quantify the effect on long-term meander migration.

This paper presents the integration of the University of Illinois RVR Meander model (Abad and García, 2006), that simulates 2D flow and morphodynamics of meandering streams, with the streambank erosion algorithms of the US Department of Agriculture-Agricultural Research Service channel evolution computer model CONCEPTS (Langendoen and Alonso, 2008; Langendoen and Simon, 2008).

A new approach is proposed, which calculates meander migration rates that are related to the physically-based streambank evolution, taking into account the processes of hydraulic erosion as well as cantilever (Figure 2a) and planar failure (Figure 2b). Mass failure mechanisms can temporarily change local bank retreat rates thereby altering migration patterns. Vertical heterogeneity of the bank and the associated differences in erodibility and shear-strength of the soils is also considered. Therefore, the proposed approach is able to yield spatially and temporally varying migration coefficients.

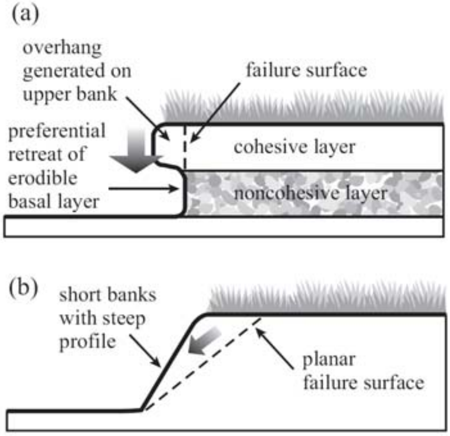

Fig. 2. (a) Cantilever failure; (b) planar failure.

Constantine et al. (2009), using data from the Sacramento River in California, showed that variability in the erodibility coefficient for hydraulic erosion explains much of the variability in the migration coefficient E_0: this confirms that the migration coefficient may be estimated directly from field data rather than from calibration, which would be especially important for streams where historical data are unavailable or controlling conditions have changed.

The computational platform that we present is a tool particularly indicated for modeling restoration and naturalization processes in rivers.

Hydrodynamics and bed morphodynamics model

The analytical solution for hydrodynamics and bed morphodynamics at a given time is derived with reference to the configuration in Figure 3 and Figure 4, with the assumptions of shallow water, small perturbation of the flow variables compared to their averaged values, and constant channel width. The model described here was first presented by Ikeda et al. (1981), and details of its derivation are found in Garcia et al. (1994) and Motta et al. (2010).

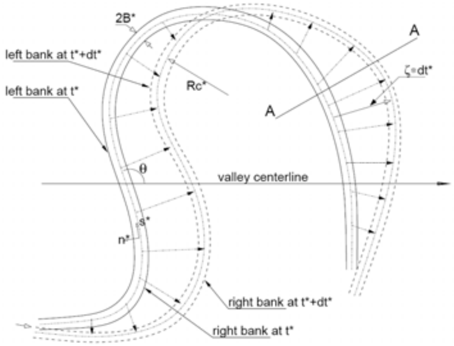

Fig. 3. Planform configuration and migration from time t^* to $t^*+\Delta t^*$.

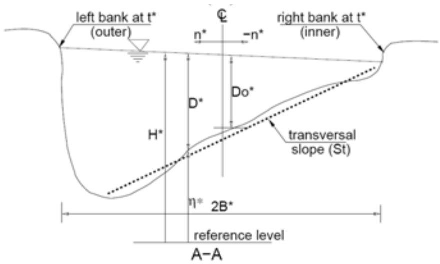

Fig. 4. Cross-section configuration.

Below are the 2D depth-averaged equations for streamwise and transverse momentum and water conservation (asterisks indicates variables with dimensions)

$$\frac{1}{1+n^*C^*}U^*\frac{\partial U^*}{\partial s^*}+V^*\frac{\partial U^*}{\partial n^*}+\frac{C^*}{1+n^*C^*}U^*V^*=-\frac{g}{1+n^*C^*}\frac{\partial H^*}{\partial s^*}-\frac{\tau_s^*}{\rho D^*} \quad (2)$$

$$\frac{1}{1+n^*C^*}U^*\frac{\partial V^*}{\partial s^*}+V^*\frac{\partial V^*}{\partial n^*}-\frac{C^*}{1+n^*C^*}U^{*2}=-g\frac{\partial H^*}{\partial n^*}-\frac{\tau_n^*}{\rho D^*} \quad (3)$$

$$\frac{1}{1+n^*C^*}\frac{\partial\left(U^*D^*\right)}{\partial s^*}+\frac{\partial\left(V^*D^*\right)}{\partial n^*}+\frac{C^*}{1+n^*C^*}V^*D^*=0 \quad (4)$$

where s^* and n^* are streamwise and transverse coordinates, U^* and V^* are streamwise and transverse depth-averaged velocities, D^* is water depth, H^* is water surface elevation, C^* is curvature, g is acceleration of gravity, ρ is water density, and τ^*_s and τ^*_n are the bed shear stress in streamwise and transverse direction, expressed as

$$\tau^*=\left(\tau_s^*,\tau_n^*\right)=\rho C_f\left(U^*,V^*\right)\sqrt{U^{*2}+V^{*2}} \quad (5)$$

where C_f is the friction coefficient calculated, according to Engelund and Hansen (1967), as

$$C_f=\left(6+2.5\ln\left(\frac{D^*}{2.5d_s^*}\right)\right)^{-2} \quad (6)$$

where d^*_s is the bed sediment particle diameter. The equations (2-4) are linearized by expressing all flow variables as the sum of a reach-averaged value (subscript “ch”) plus a perturbation (subscript “1”, which varies in both streamwise and transverse directions):

$$\begin{aligned}&\left(U^*\left(s^*,n^*\right),V^*\left(s^*,n^*\right),D^*\left(s^*,n^*\right)\right)=\\&=\left(U_{ch}^*,0,D_{ch}^*\right)+\left(U_1^*\left(s^*,n^*\right),V_1^*\left(s^*,n^*\right),D_1^*\left(s^*,n^*\right)\right)\end{aligned} \quad (7)$$

$$\begin{aligned}&\left(\tau_s^*\left(s^*,n^*\right),\tau_n^*\left(s^*,n^*\right)\right)=\\&=\left(\tau_{s,ch}^*,0\right)+\left(\tau_{s,1}^*\left(s^*,n^*\right),\tau_{n,1}^*\left(s^*,n^*\right)\right)\end{aligned} \quad (8)$$

$$C^*\left(s^*\right)=0+C_1^*\left(s^*\right) \quad (9)$$

The governing equations are then made dimensionless, by using for scaling the flow velocity U_0^* and depth D_0^* characterizing a straight channel with same width and valley slope S_0. An analytical solution is finally obtained for streamwise and transverse velocities and water depth. We report here the solution for streamwise dimensionless velocity U_1, from which the shear stress at the two banks (where transverse velocity is zero) depends:

$$U_1(s,n)=a_1(n)e^{-a_2s}+n\left(a_3C(s)+a_4e^{-a_2s}\int_0^s C(s)e^{a_2s}ds\right) \quad (10)$$

The solution (10) depends on four dimensionless parameters (contained in the expression for the coefficients a_1, a_2, a_3, and a_4): channel sinuosity, half width-to-depth ratio, Froude number squared, and channel friction coefficient. The bed morphology is described assuming that, at each cross section, the transversal bed slope is proportional to the local curvature (Figure 4), thus the model does not solve for bed elevation. No net change in sediment storage over the channel is assumed.

Streambank erosion processes

Erosion of streambanks is a combination of: (1) lateral erosion of the bank toe by hydraulic entrainment of in situ bank-materials, often termed hydraulic (or fluvial) erosion; and (2) mass failure of the upper part of the bank due to gravity (ASCE, 1998). The conceptualization and quantification of these processes in our model is briefly discussed below. Greater detail can be found in Langendoen and Alonso (2008) and Langendoen and Simon (2008).

Hydraulic erosion

The rate of hydraulic erosion of fine-grained streambank material is commonly defined by the following equation

$$E^* = k^* \left(\tau^* - \tau_c^* \right) \quad (11)$$

where k^* is the erodibility coefficient, τ^* is the shear stress acting on the bank layer and τ_c^* is its critical shear stress. The lateral erosion distance over a simulation time step Δt^* is then $E^* \Delta t^*$. An average erosion distance is computed for each layer comprising the composite bank material (Figure 6a).

Values of τ_c can be obtained from: (1) Arulanandan et al. (1980) if sodium adsorption ratio, dielectric dispersion, and pore fluid salt concentration are known; (2) in situ measurements (Hanson and Simon, 2001), as in Figures 5b,c; or (3) historical data on the retreat of the base of the bank combined with flow data. The effects of weathering processes and vegetation can be included by adjusting k^*. The predicted rate of lateral erosion is sensitive to the values used for critical shear stress and erodibility of the bank material. Critical shear stress and erodibility may vary greatly both spatially and temporally, e.g. due to variations in soil water.

Mass failure

Streambank failure occurs when gravitational forces that tend to move soil downslope exceed the forces of friction and cohesion that resist movement. The risk of failure is usually expressed by a factor of safety (FS) representing the ratio of resisting to driving forces or moments. Banks may fail by four distinct types of failure mechanisms (ASCE, 1998): (1) planar failures, (2) rotational failures, (3) cantilever failures, and (4) piping and sapping failures. Our model performs stability analyses of planar and cantilever failures, which are common failure modes of meander banks. The bank's geometry, soil properties, pore-water pressures, confining pressure exerted by the water in the stream, and riparian vegetation determine the stability of the bank.

Planar failures are analyzed using the limit equilibrium method developed for engineered slopes and embankments (e.g., Fredlund and Krahn, 1977). This method computes the required shear strength (or mobilized shear strength) to maintain a condition of limiting equilibrium. The potential failure block is divided into slices (Figure 6b), and the forces acting on them are analyzed to determine if the block remains stationary. Dividing up the soil mass into a number of slices allows one to accommodate differing slide mass geometries, stratified soils within the mass and external loads such as trees. The stability analysis in our model satisfies vertical force equilibrium for each slice and the overall horizontal force equilibrium.

Fig. 5. Field instrumentation for measuring bank properties. (a) Borehole Shear Tester (BST) for measurement of soil cohesion; (b) Jet Test for erodibility and critical shear stress; (c) Cohesive Strength Meter (CSM) for critical shear stress; (d) soil sample for density, moisture, and composition.

Integration of meander migration and bank erosion models

Channel migration is represented by the migration of its centerline. Planform characteristics (curvature, radius, orientation, etc.) are needed to use the analytical solution for calculation of hydrodynamics and bed morphodynamics illustrated earlier. The physically-based streambank erosion algorithms for hydraulic erosion, cantilever, and planar failure calculate different erosion rates of outer and inner banks at each of the centerline nodes (i.e., each cross section). Observe that the shear stress that drives the hydraulic erosion at each bank according to Eq. (11) is the one calculated with the hydrodynamic solution at $n^* = B^*$ (left bank, Figure 3) or $n^* = -B^*$ (right bank, Figure 3), and distributed along the bank profile (i.e., on the different layers) by scaling the shear stress based on either the vertical area or normal area method (Lundgren and Jonsson, 1964; Figure 6a). Planar and cantilever bank failure calculations are performed as bank profiles change over time. When FS decreases below unity, the bank material comprising the calculated failure block is removed and the bank profile is updated accordingly.

As a result of bank retreat, predicted channel width will vary in space and time. At present, two alternatives have been developed to calculate the centerline displacement at each cross section (Figure 7). In the first alternative, the bank erosion rate at each node is computed from the displacement of the stations of both the left bank and right bank toe (Figure 7a). The new channel width then varies along the channel. The width used in the hydrodynamic simulation is the minimum width along the channel. This option is suggested for relatively short-term migration scenarios.

In the second alternative, the migration distance of the channel centerline equals the physically-based retreat of the outer bank. The advance of the inner bank is assumed to equal the erosion of the outer bank for maintaining constant channel width (Figure 7b), as empirically observed in long-term migration (Ikeda et al., 1981).

Complete the computational platform: curvature filtering for spurious oscillations; Parametric Cubic Splines (PCS) for centerline-nodes regridding and addition of nodes when sinuosity increases; interpolation of bank geometry when new centerline nodes are added; optional filter for centerline smoothing in case very large curvature gradients arise (for example, for highly skewed bends). Details of all these components are found in Motta et al. (2010).

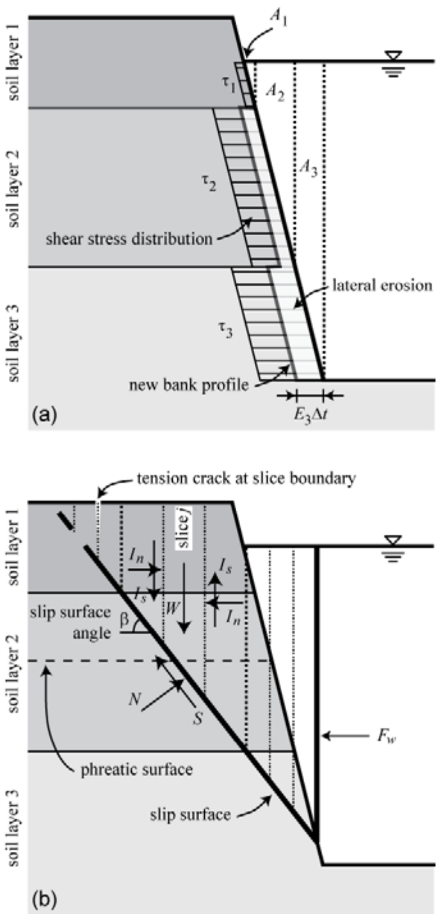

Fig. 6. Assessment of streambank erosion processes. (a) Fluvial erosion: the shown shear stress distribution is calculated using the vertical area method (Lundgren and Jonsson, 1964). (b) Mass failure: failure block configuration and forces acting on slice *j*, where $I_{n,s}$ = interslice normal and shear force, N = normal force on slice base, S = mobilized shear force on slice base, W = weight of slice, F_w = hydrostatic force exerted by the surface water on the vertical part of the slip surface, and β = failure plane angle.

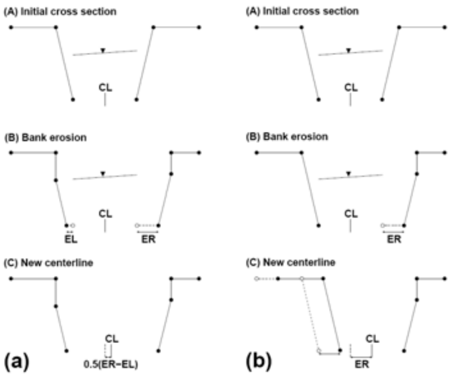

Fig. 7. Alternatives to calculate centerline migration: (a) bank retreat calculated at both inner and outer bank toes is used; and (b) only bank retreat calculated at the outer bank is used.

The RVR Meander platform is composed of different libraries (preprocessing, hydrodynamics, bank erosion, migration, filtering, plotting, and I/O). The current version of the platform is stand-alone for Windows and Linux operating systems. An interface for ArcGIS-ArcMap (Figure 8) is currently being developed.

Fig. 8. GIS interface of RVR Meander (under development).

Application

The performance of the proposed approach was tested for a reach on the Mackinaw River, Illinois, USA (Figure 9). The study reach is located in Tazewell County between the towns of South Pekin and Green Valley.

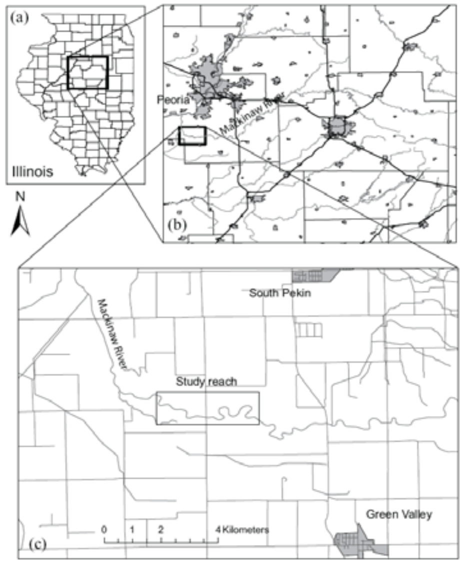

Fig. 9. Mackinaw River study reach.

The average width of the study reach is 38 m, valley slope is 0.0016, and effective discharge is 62 m^3/s. The migration of the centerline between 1951 and 1988 was simulated both with the “classic” Migration Coefficient (MC) approach in Eq. (1) and the new proposed Physically-Based (PB) approach. The dimensionless migration coefficient E_0 used for the MC method was calibrated as 3.3×10^{-7}. Bank retreat in the PB method was simulated as a combination of fluvial erosion and cantilever failures. No measurements were carried out to determine critical shear stress and erodibility coefficient. Minor calibration yielded a critical shear stress $\tau_c^* = 18$ Pa and an erodibility coefficient $k^* = 3.3 \times 10^{-8}$ m/s·Pa along the study reach.

Figure 10 compares the centerline migration obtained with MC and PB methods. The channel centerline simulated using the PB method agrees well with that observed away from the boundaries of the model reach. The channel centerline simulated using the MC method is similar to that obtained by the physically-based method for the upstream part of the study reach. However, the MC method significantly overestimates the channel centerline migration, both in terms of meander amplitude and downstream translation, along the downstream part of the study reach. In general, the PB approach shows significant improvements over the MC approach in predicting the migration of natural streams: the prediction error (ratio of the area between simulated and observed centerlines to the length of the observed centerline, which is equivalent to an average distance between simulated and observed centerlines) using the PB method is 42% smaller than that of the MC method.

Conclusions

The migration rate calculated by models of river meandering is commonly based on a method that relates the migration rate to near-bank excess velocity multiplied by a dimensionless coefficient. Its use in practice has not been successful. A new approach was presented, that relates meander migration rates to physically-based streambank evolution. The University of Illinois RVR Meander model that simulates two-dimensional flow and morphodynamics of meandering streams was integrated with the streambank erosion algorithms of the US Department of Agriculture channel evolution computer model CONCEPTS. The proposed approach for bank migration has the following advantages: (1) avoids the use of calibrated migration coefficient, (2) accounts for bank-material heterogeneity and riparian vegetation, and (3) simulates the processes controlling bank erosion. The performance of the new model was compared to that of the more simple, classic method based on the use of a migration coefficient, by simulating the evolution of a meandering reach on the Mackinaw River, Illinois, USA, showing a significantly better performance of the new approach compare to the classic approach in the less sinuous, lower part of the reach, and, in general, a significant reduction of the prediction error. More results are presented by Motta et al. (2010), which show that the new approach is able to simulate features such as high skewness and sharp necks, which are commonly observed in nature; spatial heterogeneity of floodplain soils is as important as the simulated hydrodynamics in determining the planform evolution; and mass failure mechanisms like planar failures are important but can be represented by modifying the erodibility and critical shear stress to quantify long-term migration rates. Additional testing is needed to account for spatial and temporal variations in floodplain soils (with measured properties) and vegetation to determine the performance of the proposed approach. The next application of the model will be to the Colastine River in Santa Fe, Argentina

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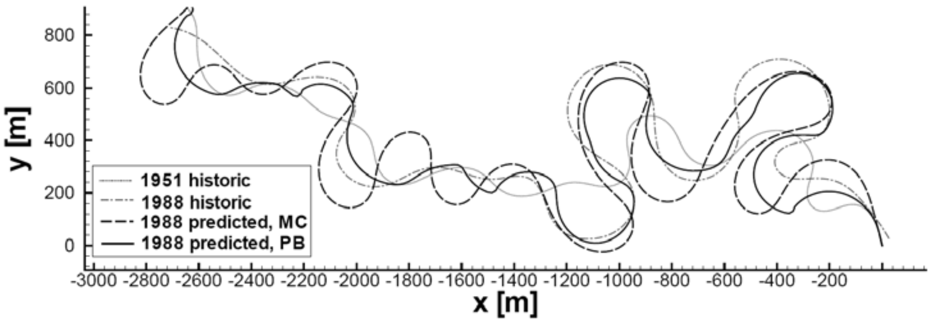

Fig. 10. Comparison between historic and simulated 1988 channel centerlines of the Mackinaw River study reach (MC: Migration Coefficient method; PB: Physically-Based Method). Flow is from right to left.

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